



Essential Instructional Content

for 2020-2021

**High School Mathematics**

This document has been adapted for use by the Nebraska Department of Education for Nebraska educators. The following guidance contains information about essential High School Mathematics content for the 2020-2021 school year.

*This guidance document is advisory in nature but binding on an agency until amended by such agency. A guidance document does not include internal procedural documents that only affect the internal operations of the agency and does not impose additional requirements or penalties on regulated parties or include confidential information or rules and regulations made in accordance with the Administrative Procedure Act. If you believe that this guidance document imposes additional requirements or penalties on regulated parties, you may request a review of the document. For comments regarding these documents contact* [*nde.guidance@nebraska.gov*](mailto:nde.guidance@nebraska.gov)*.*

*NOTE: The Nebraska version of this document was customized from a resource developed by Student Achievement Partners (SAP). The original document, 2020-2021 Priority Instructional Content in English Language Arts/Literacy and Mathematics, is located* [*here*](https://achievethecore.org/page/3267/2020-21-priority-instructional-content-in-english-language-arts-literacy-and-mathematics)*. SAP offers acknowledgements to Dr. Robin Hall, Dr. Ricki Price-Baugh, and Denise M. Walston of the Council of the Great City Schools; Phil Daro, Senior Advisor to Student Achievement Partners; as well as the teams at the Council of Chief State School Officers, the Education & Society Program at The Aspen Institute, the National Council of Teachers of Mathematics, Schoolkit, Teaching Lab, TNTP, and UnboundE*

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**Where to focus high school mathematics?**

This resource provides guidance for decisions about how to elevate some of the most important mathematics in typical high school mathematics courses in the coming school year while reducing time and intensity for topics that are less integral to the overall coherence of college- and career-ready standards.

The High School Mathematics Essential Instructional Content for the 2020–21 School Year document differs in structure from the K–8 document due primarily to the structural difference in the standards themselves: namely, that high school mathematics standards are not organized by grade level, and the ways in which states and/or districts organize standards into courses vary widely. However, similar to the K–8 document, this guidance suggests ways to reduce topics in a way that minimizes the impact to overall coherence and thereby creates some additional time in the school year for supporting students in accessing and engaging with the most important high school mathematics content. In using this guidance, decision makers should thoughtfully consider in their unique context the likely implications of the spring 2020 disruption as decisions are made to select supports to ensure that students are able to successfully engage with the course-level content. Decision makers should also bear in mind that while this document articulates content priorities, elevating the Nebraska Mathematical Processes in connection with course-level content is always a priority.

The 2020–21 school year presents a unique set of opportunities and challenges due to the disruption to instruction in spring 2020 as well as the uncertainty associated with the 2020–21 school year. The High School Mathematics Essential Instructional Content are provided in response to these conditions. They are not criteria, and they do not revise the standards. Rather, they are potential ways, and not the only ways possible, to help students engage deeply with course-level mathematics in the 2020–21 school year.

The High School Mathematics Essential Instructional Content does not stand-alone but are used in conjunction with college- and career-ready standards. One reason for this is that codes such as F-IF.A must be traced back to the standards in order to see the language to which they refer. The essential content not reiterate what the standards already say—even in cases where the specific language of a standard is fundamentally important to a high-quality aligned curriculum. Therefore, the essential content will be used most powerfully by educators who know the standards well and can use existing resources.

In constructing the recommendations for the High School Mathematics Essential Instructional Content, several resources were consulted to gain an understanding of how the standards are typically organized into courses as well as to make determinations about which standards to prioritize, which standards to de-emphasize, and which standards could reasonably be eliminated under the current circumstances. In addition to the information obtained from the resources listed below, some decisions required professional judgment of the document’s lead writers, who also serve in district roles where such guidance for the upcoming school year will be greatly needed.

Resources consulted to inform the assignment of standards to courses:

1. Utah Core Standards: Major Works (Utah State Board of Education, n.d.)
2. Achieve the Core’s High School Coherence Map (Student Achievement Partners, n.d.)
3. *Common Core State Standards for Mathematics Appendix A: Designing High School Mathematics Courses Based on the Common Core State Standards* (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010b)

Resources consulted to inform the essential content for 2020–21 school year:

1. *Common Core State Standards for Mathematics* [for standards-designated modeling] (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010a)
2. Achieve the Core’s Widely Applicable Prerequisites (Student Achievement Partners, n.d.)
3. *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (NCTM, 2018)
4. High School Core Math Content (Oregon Department of Education, in press)

For the 2020–21 school year, essential mathematical concepts and skills will support all students in meeting course-level expectations. Since the vast majority of high schools across the United States still use either an Algebra 1, Geometry, Algebra 2 sequence or some form of Integrated Mathematics I, II, and III sequence, the standards listed on the pages that follow have been coded in a way that corresponds to these courses. The tables use the following codes associated with each course: Algebra 1 (A1); Geometry (G); Algebra 2 (A2); Integrated Mathematics 1 (M1); Integrated Mathematics 2 (M2); and Integrated Mathematics 3 (M3).

**How to Read the Content Prioritization Tables**

The tables are first organized by conceptual category and cluster; then below each cluster heading, the associated standards each receive a designation to indicate the recommended level of emphasis within a particular course for the 2020–21 school year. The designations below represent the codes used to communicate this emphasis:

P - Prioritize the importance | R - Reduce the normal emphasis | E - Eliminate content to save time | -- Standard typically not taught

For standards coded with “P” for a particular course, users should interpret that to mean that no special considerations should be made for curricula well aligned to the particulars of that standard, or that the emphasis should be comparable to what it typically is for that course. Standards coded with “R” have suggestions for either reducing the emphasis on certain parts of the standard or for reducing the overall time and attention to the entire standard, or some combination of these adaptations. For these cases, there will be a note accompanying the standard to provide additional guidance related to the particular reduction in emphasis that is being suggested by the coding. Standards coded with “E” are eligible to be eliminated for the upcoming school year to make room for additional support that may be needed to ensure that students can engage successfully with the most important content of each course and to recognize that some of the modes of learning being discussed for the upcoming year simply require more time on fewer topics. The designation “--” indicates that the standard is typically taught in a different course.

One additional set of codes in the tables is designed to help users understand in part how levels of prioritization were determined. These codes are assigned to individual standards and carry the following meanings:

^ Widely Applicable Prerequisite | \* Modeling Standard | ~ Essential Concepts from Catalyzing Change

Standards that are considered “widely applicable prerequisites” are those with relatively wide applicability across a wide range of postsecondary work and often not taught for course credit in postsecondary settings. Modeling standards are those that lend themselves to developing and analyzing mathematical models for real world phenomena and generally have greater overall importance in the high school sequence of courses. Finally, standards identified as essential in *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (NCTM, 2018), are also marked as indicated above.

As a final thought, it is important to understand that these tables will not provide a one-to-one correspondence between standards and any particular scope and sequence or set of instructional materials. Well-designed mathematics curricula are structured to communicate mathematical ideas in a coherent, logical manner and often integrate standards in ways that cannot be seen when standards are shown as a list. Professional judgment, local context considerations, and flexible decision-making throughout the 2020–21 school year will be essential to effectively using the information presented on the pages that follow.

# High School Standards Prioritization Tables

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Conceptual Category: Number and Quantity**  **Domain:** **The Real Number System** | | | | | | | |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Extend the properties of exponents to rational exponents. | | | | | | | |
| HS.N.RN.A.1^~  MA 11.1.1.c | Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define 51/3 to be the cube root of 5 because we want (51/3)3 = 5(1/3)3 to hold, so (51/3)3 must equal 5*. | E | -- | P | -- | P | -- |
| HS.N.RN.A.2^~  *See Note*  MA 11.1.2.b | Rewrite expressions involving radicals and rational exponents using the properties of exponents.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | E | -- | R | -- | R | -- |
| **Cluster:** Use properties of rational and irrational numbers. | | | | | | | |
| HS.N.RN.B.3^~  MA 11.1.b | Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. | E | -- | -- | -- | E | -- |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Conceptual Category: Number and Quantity**  **Domain:** **Quantities** | | | | | | | |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Reason quantitatively and use units to solve problems.  *Note: All standards in this cluster require students to work with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.* | | | | | | | |
| HS.N.Q.A.1^~  **MA 11.3.3** | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. | P | -- | -- | P | -- | -- |
| HS.N.Q.A.2^~ | Define appropriate quantities for the purpose of descriptive modeling. | P | -- | E | P | -- | -- |
| HS.N.Q.A.3^~  MA 11.1.2.d  MA 12.3.3.b (AT) | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. | P | -- | -- | P | -- | -- |

| **Conceptual Category: Number and Quantity**  **Domain: The Complex Number System** | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Perform arithmetic operations with complex numbers. MA 11.1.2 and MA 12.1.1 | | | | | | | |
| HS.N.CN.A.1  *See Note* | Know there is a complex number *i* such that *i*2 = -1, and every complex number has the form *a + bi* with *a* and *b* real.  *Note: Combine lessons with* N.CN.C.7 *and* A.REI.B.4b *to address key concepts and reduce the amount of time spent on this standard.* | -- | -- | R | -- | R | -- |
| HS.N.CN.A.2  *See Note*  MA 11.1.1.b | Use the relation *i*2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | -- | R | -- | R | -- |
| HS.N.CN.A.3 | (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. | -- | -- | -- | -- | -- | -- |
| **Cluster:** Represent complex numbers and their operations on the complex plane. | | | | | | | |
| HS.N.CN.B.4  MA 12.1.1.a (AT) | (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. | -- | -- | -- | -- | -- | -- |
| **Cluster:** Represent complex numbers and their operations on the complex plane. (continued) MA 11.1.2 | | | | | | | |
| HS.N.CN.B.5 | (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, (-1 + √3 i)3 = 8 because (-1 + √3 i) has modulus 2 and argument 120°.* | -- | -- | -- | -- | -- | -- |
| HS.N.CN.B.6  MA 11.3.2.a | (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. | -- | -- | -- | -- | -- | -- |
| **Cluster:** Use complex numbers in polynomial identities and equations. | | | | | | | |
| HS.N.CN.C.7  *See Note*  MA 11.2.2 | Solve quadratic equations with real coefficients that have complex solutions.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | -- | R | -- | R | -- |
| HS.N.CN.C.8 | (+) Extend polynomial identities to the complex numbers. *For example, rewrite x2 + 4 as (x + 2i)(x - 2i)*. | -- | -- | E | -- | E | E |
| HS.N.CN.C.9 | (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. | -- | -- | E | -- | E | E |

Note: Vector Quantities and Matrices are not included in AGA or M1M2M3

| **Conceptual Category: Algebra**  **Domain: Seeing Structure in Expressions** | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Interpret the structure of expressions. | | | | | | | |
| HS.A-SSE.A.1 | Interpret expressions that represent a quantity in terms of its context. | | | | | | |
| HS.A-SSE.A.1a^\* | Interpret parts of an expression, such as terms, factors, and coefficients. | P | -- | P | P | P | P |
| HS.A-SSE.A.1b^\*  *See Note*  MA 11.2.2.f | Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P(1+r)2 as the product of P and a factor not depending on P*.  *Note: Reduce overall emphasis, but retain focus on interpreting expressions to shed light on a quantity in context (as described in parent standard A-SSE.A.1).* | R | -- | R | R | R | R |
| HS.A-SSE.A.2^~  *See Note*  MA 11.2.2 k. | Use the structure of an expression to identify ways to rewrite it. *For example, see x2 - y4 as (x2)2 - (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 - y2)(x2 + y2)*.  *Note: Reduce overall emphasis in earlier algebra-focused courses.* | R | -- | P | -- | R | P |
| **Cluster:** Write expressions in equivalent forms to solve problems. | | | | | | | |
| HS.A-SSE.B.3  MA 11.2.2.f | Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. | | | | | | |
| **Cluster:** Write expressions in equivalent forms to solve problems. (continued) | | | | | | | |
| HS.A-SSE.B.3a^\*  MA 11.2.2.j  MA 11.2.2.l | Factor a quadratic expression to reveal the zeros of the function it defines. | P | -- | -- | -- | P | -- |
| HS.A-SSE.B.3b^\*  *See Note*  MA 11.2.2 k. | Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic and emphasize the value of the form of the expression over fluency with the specific process of completing the square. Connect to students’ work on A-REI.B.4a.* | R | -- | -- | -- | R | -- |
| HS.A-SSE.B.3c^\*  MA 11.2.2.c | Use the properties of exponents to transform expressions for exponential functions. *For example, the expression 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%*. | P | -- | E | -- | P | -- |
| HS.A-SSE.B.4\*^  *See Note*  MA 12.1.1.f (AT) | Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*  *Note: Combine with F-BF.A.2.* | -- | -- | R | -- | -- | R |

| **Conceptual Category: Algebra**  **Domain: Arithmetic with Polynomials & Rational Expressions** | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Perform arithmetic operations on polynomials. | | | | | | | |
| HS.A-APR.A.1^  *See Note*  MA 11.2.2.i | Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.  *Note: A-APR.1 - Less emphasis on adding/subtracting and more prioritize multiplying. Combine lessons with A-SSE 2 to address key concepts and reduce the amount of time spent on this standard.* | R | -- | P | -- | R | P |
| **Cluster:** Understand the relationship between zeros and factors of polynomials. | | | | | | | |
| HS.A-.APR.B.2^  *See Note*  MA 11.2.2.l | Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p(a), so p(a) = 0 if and only if (x - a) is a factor of p(x).  *Note: Reduce overall emphasis and the number of repetitious practice problems.* | -- | -- | R | -- | -- | R |
| HS.A-APR.B.3^ | Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. | E | -- | P | -- | -- | P |
| **Cluster:** Use polynomial identities to solve problems. | | | | | | | |
| HS.A-APR.C.4^ | Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity (x2 + y2)2 = (x2 - y2)2 + (2xy)2 can be used to generate Pythagorean triples.* | -- | -- | E | -- | -- | E |
| **Cluster:** Use polynomial identities to solve problems. (continued) | | | | | | | |
| HS.A-APR.C.5^  MA 11.2.2 k. | (+) Know and apply the Binomial Theorem for the expansion of (*x* + *y*)*n* in powers of *x* and *y* for a positive integer *n*, where *x* and *y* are any numbers, with coefficients determined for example by Pascal's Triangle. | -- | -- | E | -- | -- | E |
| **Cluster:** Rewrite rational expressions. | | | | | | | |
| HS.A-APR.D.6^  *See Note*  MA 11.2.2.d | Rewrite simple rational expressions in different forms; write *a*(*x*)/*b*(*x*) in the form *q*(*x*) + *r*(*x*)/*b*(*x*), where *a*(*x*), *b*(*x*), *q*(*x*), and *r*(*x*) are polynomials with the degree of *r*(*x*) less than the degree of *b*(*x*), using inspection, long division, or, for the more complicated examples, a computer algebra system.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic. Connect to A-APR.B.2.* | -- | -- | R | -- | -- | R |
| HS.A-APR.D.7^  MA 11.2.2.i | (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. | -- | -- | E | -- | -- | E |

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| --- | --- | --- | --- | --- | --- | --- | --- |
| **Conceptual Category: Algebra**  **Domain: Creating Equations** | | | | | | | |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Create equations that describe numbers or relationships. | | | | | | | |
| HS.A-CED.A.1^\*  MA 11.2.2.f | Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*. | P | -- | P | P | P | P |
| HS.A-CED.A.2^\*  MA 11.2.2.h | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | P | -- | P | P | P | P |
| HS.A-CED.A.3^\*  MA 11.2.2.b | Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods*. | P | -- | P | P | -- | P |
| HS.A-CED.A.4^\*  *See Note*  MA 11.2.2.f | Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R*.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | P | -- | P | P | P | R |

| **Conceptual Category: Algebra**  **Domain:** **Reasoning with Equations and Inequalities** | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Understand solving equations as a process of reasoning and explain the reasoning. | | | | | | | |
| HS.A-REI.A.1^~  *See Note*  MA 11.2.2.b | Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.  *Note: Lessen the normal emphasis on problem types related to explaining each step and elevate the importance of constructing viable arguments.* | R | -- | E | R | -- | -- |
| HS.A-REI.A.2^  MA 11.2.2.g | Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. | -- | -- | P | -- | -- | P |
| **Cluster:** Solve equations and inequalities in one variable. | | | | | | | |
| HS.A-REI.B.3^  *See Note*  MA 11.2.2.g | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | R | -- | -- | R | -- | -- |
| HS.A-REI.B.4  MA 11.2.1.g  MA 11.2.2.n | Solve quadratic equations in one variable. | | | | | | |
| **Cluster:** Solve equations and inequalities in one variable. (continued) | | | | | | | |
| HS.A-REI.B.4a^  *See Note* | Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form (*x* - *p*)2 = *q* that has the same solutions. Derive the quadratic formula from this form.  *Note: Lessen the normal emphasis on deriving the quadratic formula and reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | R | -- | -- | -- | R | -- |
| HS.A-REI.B.4b^~  *See Note* | Solve quadratic equations by inspection (e.g., for *x*2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as *a* ± *bi* for real numbers *a* and *b*.  *Note: Lessen the emphasis on completing the square and emphasize solving by inspection, taking square roots, quadratic formula, and factoring; recognize when quadratic formula gives non-real solutions but reduce emphasis on this case.* | R | -- | R | -- | R | -- |
| **Cluster:** Solve systems of equations. | | | | | | | |
| HS.A-REI.C.5^ | Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. | E | -- | -- | E | -- | -- |
| **Cluster:** Solve systems of equations. (continued) | | | | | | | |
| HS.A-REI.C.6^  MA 11.2.2.h | Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. | P | -- | E | P | -- | -- |
| HS.A-REI.C.7^  *See Note* | Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line *y* = -3*x* and the circle *x*2 + *y2* = 3.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | R | -- | E | -- | R | -- |
| HS.A-REI.C.8  MA 12.3.2.d (AT) | (+) Represent a system of linear equations as a single matrix equation in a vector variable. | -- | -- | -- | -- | -- | -- |
| HS.A-REI.C.9 | (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). | -- | -- | -- | -- | -- | -- |
| **Cluster:** Represent and solve equations and inequalities graphically. | | | | | | | |
| HS.A-REI.D.10^  MA 11.2.2.h | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). | P | -- | -- | P | -- | -- |
| **Cluster:** Represent and solve equations and inequalities graphically. (continued) | | | | | | | |
| HS.A-REI.D.11^\*~  MA 11.2.1.e  MA 11.2.1.f | Explain why the *x*-coordinates of the points where the graphs of the equations *y* = *f*(*x*) and *y* = *g*(*x*) intersect are the solutions of the equation *f*(*x*) = *g*(*x*); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where *f*(*x*) and/or *g*(*x*) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. | P | -- | P | P | -- | P |
| HS.A-REI.D.12^~  *See Note*  MA 11.2.1.e  MA 11.2.1.f | Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.  *Note: Emphasize problems that ground the mathematics in real world contexts.* | P | -- | -- | P | -- | -- |

| **Conceptual Category: Functions**  **Domain: Interpreting Functions** | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Understand the concept of a function and use function notation. | | | | | | | |
| HS.F-IF.A.1^~  MA 11.2.1.d | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then *f*(*x*) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation *y* = *f*(*x*). | P | -- | -- | P | -- | -- |
|
| HS.F-IF.A.2^  MA 11.2.1.a | Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. | P | -- | -- | P | -- | -- |
| HS.F-IF.A.3^  *See Note* | Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1*.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | R | -- | R | R | -- | -- |
| **Cluster:** Interpret functions that arise in applications in terms of the context.  *M1 - Linear, exponential, and quadratic*  *M2 - Emphasize selection of appropriate models* | | | | | | | |
| HS.F-.IF.B.4^\*~  **MA 11.2.1** | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums;*  *symmetries; end behavior; and periodicity*. | P | -- | P | P | P | P |
| **Cluster:** Interpret functions that arise in applications in terms of the context. (continued)  *M1 - Linear, exponential, and quadratic*  *M2 - Emphasize selection of appropriate models* | | | | | | | |
| HS.F-IFB.5^\* | Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*\* | P | -- | P | P | P | P |
| HS.F-IF.B.6^\*  MA 11.2.2.e | Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. | P | -- | P | P | P | P |
| **Cluster:** Analyze functions using different representations. | | | | | | | |
| HS.F-IF.C.7 | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | | | | | | |
| HS.F-IF.C.7a^\*  MA 11.2.1.g MA 12.3.2.a (AT) | Graph linear and quadratic functions and show intercepts, maxima, and minima. | P | -- | -- | P | P | -- |
| HS.F-IF.C.7b^\*  *See Note*  MA 12.2.1.a (AT) | Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.  *Note: Eliminate step functions; emphasize square root and cube root.* | R | -- | P | -- | P | R |
| HS.F-IF.C.7c^\* | Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. | -- | -- | P | -- | -- | P |
| **Cluster:** Analyze functions using different representations. (continued) | | | | | | | |
| HS.F-IF.C.7d | (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. | -- | -- | -- | -- | -- | -- |
| HS.F-IF.C.7e^\*  MA 12.2.1.a (AT) | Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. | P | -- | P | P | -- | P |
| HS.F-IF.C.8 | Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. | | | | | | |
| HS.F-IF.C.8a^  *See Note*  MA 12.3.2.b (AT) | Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.  *Note: Reduce the number of repetitious practice problems related to factoring trinomials over the integers, and emphasize using the factored form to draw conclusions. Connect to* HS.A-SSE.B.3b. | R | -- | R | -- | R | -- |
| HS.F-IF.C.8b^  MA 11.2.3.a | Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as y = (1.02)ᵗ, y = (0.97)ᵗ, y = (1.01)12ᵗ, y = (1.2)t/10, and classify them as representing exponential growth or decay.* | E | -- | E | -- | E | -- |
| **Cluster:** Analyze functions using different representations. (continued) | | | | | | | |
| HS.F-IF.C.9^  *See Note*  MA 11.2.1  MA 11.2.1.h  MA 11.2.3.a | Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum*.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | P | -- | R | P | P | P |

| **Conceptual Category: Functions**  **Domain: Building Functions** | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Build a function that models a relationship between two quantities. | | | | | | | |
| HS.F-BF.A.1  MA 11.2.3.a MA 11.2.1.a | Write a function that describes a relationship between two quantities. | | | | | | |
|
| HS.F-BF.A.1a^\*  *See Note* | Determine an explicit expression, a recursive process, or steps for calculation from a context.  *Note: Combine with F-BF.A.2, F-LE.A.2 and F-IF.A.3 to address key concepts and reduce the amount of time spent on this standard.* | R | -- | E | R | R | E |
| HS.F-BF.A.1b^\*  MA 11.2.2.m  .2.1.d (AT) | Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model*. | E | -- | E | E | E | E |
| HS.F-BF.A.1c  .2.1.d (AT) | (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time*. | -- | -- | -- | -- | -- | -- |
| HS.F-BF.A.2\*  *See Note*  MA 12.1.1.f (AT) | Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.  *Note: Combine with F-BF.A.1b and F-LE.A.2 to address key concepts and reduce the amount of time spent on this standard.* | R | -- | R | R | -- | -- |
| **Cluster:** Build new functions from existing functions. | | | | | | | |
| HS.F-BF.B.3  MA 12.2.1.d (AT) | Identify the effect on the graph of replacing *f*(*x*) by *f*(*x*) + *k*, *k* *f*(*x*), *f*(*kx*), and *f*(*x* + *k*)for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. | P | -- | P | P | P | P |
| HS.F-BF.B.4  MA 11.2.1.h | Find inverse functions. *For example, f(x) = or f(x) = (x+1)/(x-1) for x ≠ 1*. | | | | | | |
| HS.F-BF.B.4a  *See Note*  MA 11.2.1.h | Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | E | -- | R | -- | E | R |
| HS.F-BF.B.4b  *See Note*  MA 11.2.1.h | (+) Verify by composition that one function is the inverse of another.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | -- | -- | -- | -- | R |
| HS.F-BF.B.4c  MA 11.2.1.h | (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. | -- | -- | -- | -- | E | E |
| HS.F-BF.B.4d | (+) Produce an invertible function from a non-invertible function by restricting the domain. | -- | -- | -- | -- | E | E |
| HS.F-BF.B.5  MA 11.2.1.h | (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. | -- | -- | -- | -- | -- | -- |

| **Conceptual Category: Functions**  **Domain: Linear, Quadratic, and Exponential** | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Construct and compare linear, quadratic, and exponential models and solve problems. | | | | | | | |
| HS.F-LE.A.1 | Distinguish between situations that can be modeled with linear functions and with exponential functions. | | | | | | |
|
| HS.F-LE.A.1a^\*~ | Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. | P | -- | -- | P | -- | -- |
| HS.F-LE.A.1b^\*~  MA 11.2.3.a | Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. | P | -- | -- | P | -- | -- |
| HS.F-LE.A.1c^\*~  MA 11.2.3.a | Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. | P | -- | -- | P | -- | -- |
| HS.F-LE.A.2\*~  MA 11.2.3.a | Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). | P | -- | E | P | -- | -- |
| HS.F-LE.A.3\*  *See Note*  MA 11.2.3.a | Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.  *Note: Combine with F-LE.A.1b and F-LE.A.1c to address key concepts and reduce the amount of time spent on this standard.* | R | -- | -- | R | R | R |
| **Cluster:** Construct and compare linear, quadratic, and exponential models and solve problems. (continued) | | | | | | | |
| HS.F-LE.A.4\*  *See Note*  MA 11.2.3.a | For exponential models, express as a logarithm the solution to *abct* = *d* where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | -- | R | -- | -- | R |
| **Cluster:** Interpret expressions for functions in terms of the situation they model. | | | | | | | |
| HS.F-LE.B.5\*~  MA 11.2.3.a | Interpret the parameters in a linear or exponential function in terms of a context. | P | -- | E | P | -- | -- |

| **Conceptual Category: Functions**  **Domain: Trigonometric Functions** | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Extend the domain of trigonometric functions using the unit circle. | | | | | | | |
| HS.F-TF.A.1  MA 12.2.1.f (AT)  MA 12.2.1.g (AT) | Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. | -- | -- | P | -- | -- | P |
|
| HS.F-TF.A.2  MA 12.2.1.b (AT) | Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. | -- | -- | P | -- | -- | P |
| HS.F-TF.A.3  MA 12.2.1.c (AT) | (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for *x*, π + *x*, and 2π - *x* in terms of their values for *x*, where *x* is any real number. | -- | -- | -- | -- | -- | -- |
| HS.F-TF.A.4  MA 12.2.2.b (AT) | (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. | -- | -- | -- | -- | -- | -- |
| **Cluster:** Model periodic phenomena with trigonometric functions. | | | | | | | |
| HS.F-TF.B.5\*  MA 12.2.2.d (AT)  MA 12.2.3.a (AT) | Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. | -- | -- | P | -- | -- | P |
| HS.F-TF.B.6  MA 12.2.2.c (AT) | (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. | -- | -- | -- | -- | -- | -- |
| **Cluster:** Model periodic phenomena with trigonometric functions. (continued) | | | | | | | |
| HS.F-TF.B.7  MA 11.2.1.h  MA 11.2.3.a  MA 12.2.3.b (AT) | (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. | -- | -- | -- | -- | -- | -- |
| **Cluster:** Prove and apply trigonometric identities. | | | | | | | |
| HS.F-TF.C.8  MA 12.2.2.a (AT) | Prove the Pythagorean identity(θ) + (θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. | -- | -- | E | -- | E | E |
| HS.F-TF.C.9 | (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. | -- | -- | -- | -- | -- | -- |

| **Conceptual Category: Geometry**  **Domain: Congruence** | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Experiment with transformations in the plane. | | | | | | | |
| HS.G-CO.A.1^  *See Note*  MA 11.3.1.a | Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.  *Note: Combine with G-CO.A.4 to address key concepts and reduce the amount of time spent on this standard.* | -- | R | -- | R | -- | -- |
|
| HS.G-CO.A.2^~  MA 11.3.1.h | Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). | -- | P | -- | P | -- | -- |
| HS.G-CO.A.3^~  *See Note*  MA 11.3.2.h  MA 11.3.2.i | Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.  *Note: Combine with G-CO.A.2 to address key concepts and reduce the amount of time spent on the standard.* | -- | R | -- | R | -- | -- |
| HS.G-CO.A.4^ | Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. | -- | P | -- | P | -- | -- |
| HS.G-CO.A.5^~ MA 11.3.2.g  MA 11.3.2.h  MA 11.3.2.i  MA 11.3.2.j | Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. | -- | P | -- | P | -- | -- |
| **Cluster:** Understand congruence in terms of rigid motions. | | | | | | | |
| HS.G-CO.B.6^~  MA 8.3.2.a | Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. | -- | P | -- | P | -- | -- |
| HS.G-.CO.B.7^~ | Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. | -- | P | -- | P | -- | -- |
| HS.G-CO.B.8^ | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. | -- | P | -- | P | -- | -- |
| **Cluster:** Prove geometric theorems. | | | | | | | |
| HS.G-CO.C.9^~  MA 8.3.1.b | Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints*. | -- | P | -- | -- | P | -- |
| HS.G-CO.C.10^~  *See Note*  MA 8.3.1.a | Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point*.  *Note: Reduce overall time spent on proving theorems.* | -- | R | -- | -- | R | -- |
| **Cluster:** Prove geometric theorems. (continued) | | | | | | | |
| HS.G-CO.C.11  *See Note*  MA 11.3.1.b | Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals*.  *Note: Reduce overall time spent on proving theorems.* | -- | R | -- | -- | R | -- |
| **Cluster:** Make geometric constructions. MA 11.3.1.h | | | | | | | |
| HS.G-CO.D.12  MA 11.3.1.h | Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line*. | -- | P | -- | P | -- | -- |
| HS.G-CO.D.13  MA 11.3.1.h | Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. | -- | E | -- | E | -- | -- |

| **Conceptual Category: Geometry**  **Domain: Similarity, Right Triangles, and Trigonometry** | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Understand similarity in terms of similarity transformations. | | | | | | | |
| HS.G-SRT.A.1  MA 11.3.2.j | Verify experimentally the properties of dilations given by a center and a scale factor: | | | | | | |
|
| HS.G-SRT.A.1a^  MA 11.3.2.j | A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. | -- | P | -- | -- | P | -- |
| HS.G-SRT.A.1b^  *See Note* MA 11.3.3.c | The dilation of a line segment is longer or shorter in the ratio given by the scale factor.  *Note: Combine with students’ work on G-SRT.A.1a.* | -- | R | -- | -- | R | -- |
| HS.G-SRT.A.2^~  MA 8.3.2.c | Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. | -- | P | -- | -- | P | -- |
| HS.G-SRT.A.3^  MA 11.3.1.c | Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. | -- | P | -- | -- | P | -- |
| **Cluster:** Prove theorems involving similarity. | | | | | | | |
| HS.G-SRT.B.4^  MA 8.3.3.a  MA 11.3.1.d | Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* | -- | P | -- | -- | P | -- |
| HS.G-SRT.B.5^  MA 11.3.1.c | Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. | -- | P | -- | -- | P | -- |
| **Cluster:** Define trigonometric ratios and solve problems involving right triangles. | | | | | | | |
| HS.G-SRT.C.6^\* | Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. | -- | P | -- | -- | P | -- |
| HS.G-.SRT.C.7  *See Note* | Explain and use the relationship between the sine and cosine of complementary angles.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | -- | -- | R | -- |
| HS.G-SRT.C.8\*~  *See Note*  MA 8.3.3.b  MA 11.3.1.e | Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.  *Note:* *Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | -- | -- | R | -- |
| **Cluster:** Apply trigonometry to general triangles. | | | | | | | |
| HS.G-SRT.D.9 | (+) Derive the formula *A* = 1/2 *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. | -- | E | -- | -- | -- | E |
| HS.G-SRT.D.10 ^  *See Note* | (+) Prove the Laws of Sines and Cosines and use them to solve problems.  *Note: Lessen the normal emphasis on proofs and elevate the importance of solving problem types.* | -- | E | -- | -- | -- | R |
| HS.G-SRT.D.11^  *See Note*  MA 12.3.1.a (AT) | (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | E | -- | -- | -- | R |

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| **Conceptual Category: Geometry**  **Domain: Circles** | | | | | | | |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Understand and apply theorems about circles. | | | | | | | |
| HS.G-C.A.1 | Prove that all circles are similar. | -- | E | -- | -- | E | -- |
|
| HS.G-C.A.2  *See Note*  MA 11.3.1.f  MA 11.3.1.g  MA 12.3.1.b (AT) | Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*  *Note: Emphasize primarily the concept of perpendicularity between the radius and any tangent to the circle.* | -- | R | -- | -- | R | R |
| HS.G-C.A.3  MA 11.3.1.h | Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. | -- | E | -- | -- | E | -- |
| HS.G-C.A.4  MA 11.3.1.h | (+) Construct a tangent line from a point outside a given circle to the circle. | -- | E | -- | -- | E | -- |
| **Cluster:** Find arc lengths and areas of sectors of circles. MA 11.3.3.d | | | | | | | |
| HS.G-C.B.5  *See Note* | Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.  *Note: Reduce overall emphasis on the standard but retain the core definition of radian measure as described in the standard.* | -- | R | -- | -- | R | R |

| **Conceptual Category: Geometry**  **Domain: Expressing Geometric Properties with Equations** | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Translate between the geometric description and the equation for a conic section. | | | | | | | |
| HS.G-GPE.A.1  MA 11.3.2.k | Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. | -- | P | -- | -- | P | -- |
|
| HS.G-GPE.A.2 MA 12.3.2.f (AT) | Derive the equation of a parabola given a focus and directrix. | -- | E | E | -- | E | -- |
| HS.G-GPE.A.3  MA 12.3.2.f (AT) | (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. | -- | -- | -- | -- | -- | E |
| **Cluster:** Use coordinates to prove simple geometric theorems algebraically. | | | | | | | |
| HS.G-GPE.B.4~  MA 11.3.2.e MA 11.3.2.f | Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* | -- | P | -- | P | P | -- |
| HS.G-GPE.B.5~  *See Note*  MA 11.3.2.b  MA 11.3.2.c | Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | -- | R | -- | -- |
| **Cluster:** Use coordinates to prove simple geometric theorems algebraically. (continued) | | | | | | | |
| HS.G-GPE.B.6  MA 11.3.2.a  MA 12.3.2.c (AT) | Find the point on a directed line segment between two given points that partitions the segment in a given ratio. | -- | E | -- | -- | E | -- |
| HS.G-GPE.B.7\*  *See Note*  MA 11.3.2.d  MA 11.3.3.e | Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.  *Note: Emphasize understanding the formula conceptually, use it to solve real world problems, and reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | -- | R | -- | -- |

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| --- | --- | --- | --- | --- | --- | --- | --- |
| **Conceptual Category: Geometry**  **Domain: Geometric Measurement and Dimension** | | | | | | | |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Explain volume formulas and use them to solve problems. | | | | | | | |
| HS.G-GMD.A.1 | Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments*. | -- | E | -- | -- | E | -- |
|
| HS.G-GMD.A.2  MA 12.3.3.a (AT) | (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. | -- | E | -- | -- | -- | -- |
| HS.G-GMD.A.3\*~  MA 8.3.3.d  MA 11.3.3.e | Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. | -- | P | -- | -- | P | -- |
| **Cluster:** Visualize relationships between two-dimensional and three-dimensional objects. | | | | | | | |
| HS.G-GMD.B.4  *See Note*  MA 11.3.3.c  MA 12.3.2.g (AT)  MA 12.3.2.h (AT) | Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | -- | -- | -- | R |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Conceptual Category: Geometry**  **Domain: Modeling with Geometry** | | | | | | | |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Apply geometric concepts in modeling situations. | | | | | | | |
| HS.G-MG.A.1\*~ | Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).\* | -- | P | -- | -- | -- | P |
|
| HS.G-MG.A.2\*~  MA 11.3.3.a | Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).\* | -- | P | -- | -- | -- | P |
| HS.G-MG.A.3\*~ | Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).\* | -- | P | -- | -- | -- | P |

| **Conceptual Category: Statistics & Probability**  **Domain: Interpreting Categorical and Quantitative Data** | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Summarize, represent, and interpret data on a single count or measurement variable. | | | | | | | |
| HS.S-ID.A.1\*~ | Represent data with plots on the real number line (dot plots, histograms, and box plots). | E | -- | -- | E | -- | -- |
|
| HS.S-ID.A.2^\*~  MA 11.4.2.a | Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. | P | -- | -- | P | -- | -- |
| HS.S-ID.A.3\*~  MA 11.4.2.b  MA 11.4.2.f | Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). | P | -- | -- | P | -- | -- |
| HS.S-ID.A.4\*~  MA 11.4.2.k | Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. | -- | -- | P | -- | -- | P |
| **Cluster:** Summarize, represent, and interpret data on two categorical and quantitative variables. | | | | | | | |
| HS.S-ID.B.5\*~  MA 11.4.2.c | Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. | P | -- | -- | P | -- | -- |
| **Cluster:** Summarize, represent, and interpret data on two categorical and quantitative variables. (continued) | | | | | | | |
| HS.S-ID.B.6 | Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. | | | | | | |
| HS.S-ID.B.6a\*~ | Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. | P | -- | E | P | -- | -- |
| HS.S-ID.B.6b\*~ | Informally assess the fit of a function by plotting and analyzing residuals. | P | -- | -- | P | -- | -- |
| HS.S-ID.B.6c\*~  MA 11.4.2.i | Fit a linear function for a scatter plot that suggests a linear association. | P | -- | -- | P | -- | -- |
| **Cluster:** Interpret linear models. | | | | | | | |
| HS.S-ID.C.7^\*~  MA 8.2.1.b | Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. | P | -- | -- | P | -- | -- |
| HS.S-ID.C.8\*~  *See Note*  MA 11.4.2.e | Compute (using technology) and interpret the correlation coefficient of a linear fit.  *Note: Emphasize interpreting the correlation coefficient.* | R | -- | -- | R | -- | -- |
| HS.S-ID.C.9\*~  MA 11.4.2.j | Distinguish between correlation and causation. | P | -- | -- | P | -- | -- |

| **Conceptual Category: Statistics & Probability**  **Domain: Making Inferences and Justifying Conclusions** | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Understand and evaluate random processes underlying statistical experiments. | | | | | | | |
| HS.S-IC.A.1^\*~ | Understand statistics as a process for making inferences about population parameters based on a random sample from that population. | -- | -- | P | -- | -- | P |
|
| HS.S-IC.A.2^\*~  MA 11.4.2.g | Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model*? | -- | -- | P | -- | -- | P |
| **Cluster:** Make inferences and justify conclusions from sample surveys, experiments, and observational studies. MA 11.4.2.d MA 12.4.2.a (AT) | | | | | | | |
| HS.S-IC.B.3\*~  *See Note*  MA 11.4.2.h | Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.  *Note: Combine lessons with S-IC.B.4 and S-IC.B.5 to address key concepts and reduce the amount of time spent on this standard. Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | -- | R | -- | -- | R |
| HS.S-IC.B.4\*~  *See Note* | Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.  *Note: Combine lessons with S-IC.B.3 and S-IC.B.5 to address key concepts and reduce the amount of time spent on this standard. Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | -- | R | -- | -- | R |
| **Cluster:** Make inferences and justify conclusions from sample surveys, experiments, and observational studies. (continued) MA 12.4.2.a (AT) | | | | | | | |
| HS.S-IC.B.5\*~  *See Note* | Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.  *Note: Combine lessons with S-IC.B.3 and S-IC.B.4 to address key concepts and reduce the amount of time spent on this standard. Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | -- | R | -- | -- | R |
| HS.S-IC.B.6\*~  *See Note* | Evaluate reports based on data.  *Note: Reduce the normal emphasis.* | -- | -- | R | -- | -- | R |

| **Conceptual Category: Statistics & Probability**  **Domain: Conditional Probability and the Rules of Probability** | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Understand independence and conditional probability and use them to interpret data. | | | | | | | |
| HS.S-.CP.A.1\*~ | Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). | -- | P | E | -- | P | -- |
|
| HS.S-.CP.A.2\*~  *See Note* | Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.  *Note: Combine with lessons on other S-CP.A standards to address key concepts and reduce the amount of time spent on this standard. Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | E | -- | R | -- |
| HS.S-CP.A.3\*~  *See Note* | Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.  *Note: Combine with lessons on other S-CP.A standards to address key concepts and reduce the amount of time spent on this standard. Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | E | -- | R | -- |
| **Cluster:** Understand independence and conditional probability and use them to interpret data. (continued) | | | | | | | |
| HS.S-CP.A.4\*~ | Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.* | -- | P | E | -- | P | -- |
| HS.S-CP.A.5\*~ | Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.* | -- | P | E | -- | P | -- |
| **Cluster:** Use the rules of probability to compute probabilities of compound events. MA 11.4.3.c | | | | | | | |
| HS.S-CP.B.6\*  *See Note* | Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | E | -- | R | -- |
| HS.S-CP.B.7\*  *See Note* | Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.  *Note: Reduce the number of repetitious practice problems that would normally be assigned to students for this topic.* | -- | R | E | -- | R | -- |
| **Cluster:** Use the rules of probability to compute probabilities of compound events. (continued) | | | | | | | |
| HS.S-CP.B.8\* | (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. | -- | E | E | -- | E | -- |
| HS.S-CP.B.9\*  MA 11.4.3.b | (+) Use permutations and combinations to compute probabilities of compound events and solve problems. | -- | E | E | -- | E | -- |

| **Conceptual Category: Statistics & Probability**  **Domain: Using Probability to Make Decisions** | | | | | | | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Standard** | **Language of Standard** | **Courses** | | | | | |
| A1 | G | A2 | M1 | M2 | M3 |
| **Cluster:** Calculate expected values and use them to solve problems. | | | | | | | |
| HS.S-MD.A.1  MA 11.4.3.a | (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. | -- | -- | -- | -- | -- | -- |
|
| HS.S-MD.A.2  MA 12.4.3.a (AT) | (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. | -- | -- | -- | -- | -- | -- |
| HS.S-MD.A.3  MA 11.4.3.a | (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.* | -- | -- | -- | -- | -- | -- |
| HS.S-MD.A.4  MA 11.4.3.a | (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?* | -- | -- | -- | -- | -- | -- |
| **Cluster:** Use probability to evaluate outcomes of decisions. | | | | | | | |
| HS.S-MD.B.5  MA 12.4.3.b (AT) | (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. | | | | | | |
| HS.S-MD.B.5a | Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.* | -- | -- | -- | -- | -- | -- |
| HS.S-MD.B.5b  MA 12.4.3.c (AT) | Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.* | -- | -- | -- | -- | -- | -- |
| HS.S-MD.B.6 | (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). | -- | E | E | -- | E | E |
| HS.S-MD.B.7  MA 12.4.3.d (AT) | (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). | -- | E | E | -- | E | E |

## Additional Resources

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## Social, Emotional, and Academic Development (SEAD) Sources

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## Resources Consulted to Inform the Assignment of Standards to Courses and the Prioritization for the 2020–21 School Year

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Oregon Department of Education. (in press). *High school core math content*.

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