DISRUPTION & INNOVATION

Assessing and Diagnosing Unfinished Learning in Math

Erin McCopp

February 24, 2021

NEBRASKA DEPARTMENT OF EDUCATION

Zoom Norms

- **Be present:** keep camera on when possible
- **Audio:** stay on "mute" if you are not speaking
- **Engage with others:** Zoom Breakout Rooms feature!
- **Chat:** use the chat feature when prompted
- **Materials**: Soft copy of the note catcher; links in chat box

Learning Series at a Glance

Session 1	Session 2	Session 3	Session 4
Defining our Approach to Addressing Unfinished Teaching and Learning in Math	Assessing and Diagnosing Unfinished Learning in Math	Plan and Take Action Part I: Planning Intentional Core Supports	Plan and Take Action Part II: Planning Intentional Small Group and Individual Supports

Which image best captures your experience in K-12 math?

1

2

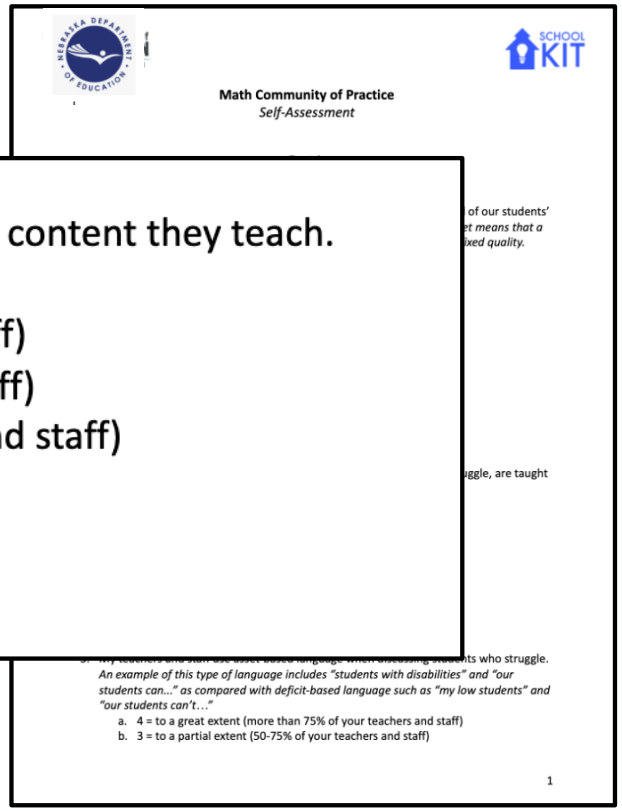
3

4

Self-Assessment Reflection

2. My teachers and support staff have a strong knowledge of the math content they teach.
    a. 4 = to a great extent (more than 75% of the time)
    b. 3 = to a partial extent (50-75% of your teachers and staff)
    c. 2 = to a limited extent (25-49% of your teachers and staff)
    d. 1 = little to no extent (less than 25% of your teachers and staff)

Provide evidence for your response.



Math Community of Practice
Self-Assessment

An example of this type of language includes "students with disabilities" and "our students can..." as compared with deficit-based language such as "my low students" and "our students can't…"

a. 4 = to a great extent (more than 75% of your teachers and staff)
b. 3 = to a partial extent (50-75% of your teachers and staff)

1

What are we doing today? Why?

UNDERSTAND.

DIAGNOSE.

PLAN &
TAKE ACTION.

Access Materials

Access today's Note Catcher at the following link:

https://tinyurl.com/NDESession2MaterialsK-5

Session Agenda

Time	Topic
15 min	Getting Started
25 min	Deepening Understanding of the Math We Teach
40 min	Diagnosing Unfinished Learning
10 min	Reflection & Wrapping Up

Our Agenda

1. Getting Started
2. Deepening Understanding of the Math We Teach
3. Assessing and Diagnosing Unfinished Learning
4. Reflection & Wrapping Up

Understanding is NOT an ON/OFF switch

"Possessing deep knowledge of mathematical content means that teachers can pose good problems, ask good questions, and guide students to understanding by knowing where they want students to be."
(Taper, 2012)

Preparing to Teach

4.NF.A.2 (4.1.1.k):

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)

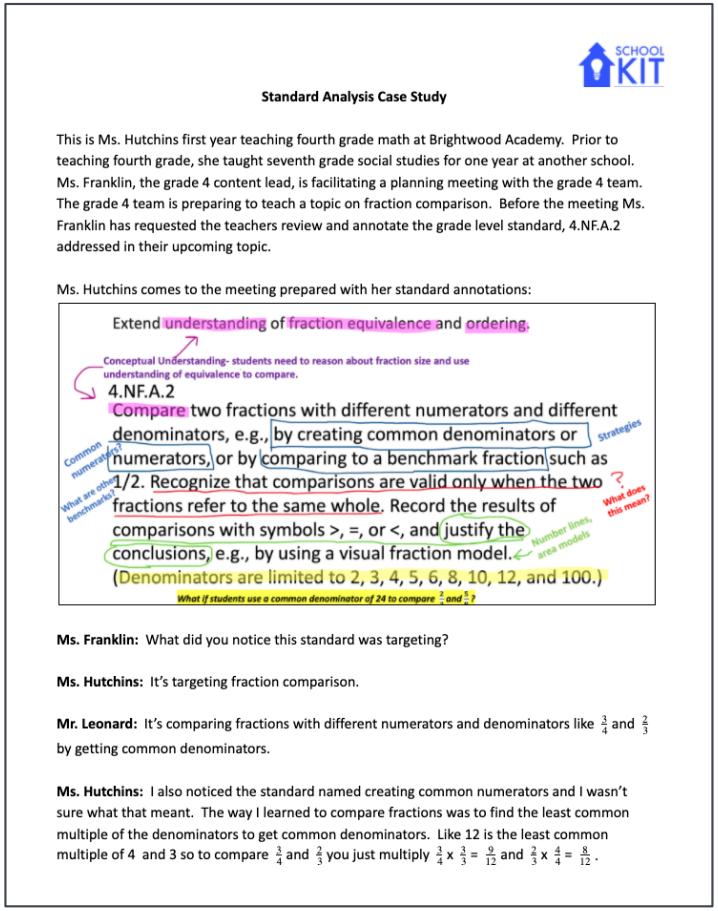
Ms. Hutchins
Brightwood Academy
Grade 4 Teacher

Uncovering the Standards

Review Ms. Hutchins Standards Analysis

Reflect:

- What do you notice?
- What do you wonder?



SCHOOL KIT

Standard Analysis Case Study

This is Ms. Hutchins first year teaching fourth grade math at Brightwood Academy. Prior to teaching fourth grade, she taught seventh grade social studies for one year at another school. Ms. Franklin, the grade 4 content lead, is facilitating a planning meeting with the grade 4 team. The grade 4 team is preparing to teach a topic on fraction comparison. Before the meeting Ms. Franklin has requested the teachers review and annotate the grade level standard, 4.NF.A.2 addressed in their upcoming topic.

Ms. Hutchins comes to the meeting prepared with her standard annotations:

Extend understanding of fraction equivalence and ordering.

Conceptual Understanding- students need to reason about fraction size and use understanding of equivalence to compare.

4.NF.A.2

Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

(Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)

Common numerators

What are other benchmarks?

Strategies

Number lines, area models

What does this mean?

What if students use a common denominator of 24 to compare $\frac{7}{8}$ and $\frac{5}{6}$?

Ms. Franklin: What did you notice this standard was targeting?

Ms. Hutchins: It's targeting fraction comparison.

Mr. Leonard: It's comparing fractions with different numerators and denominators like $\frac{3}{4}$ and $\frac{2}{3}$ by getting common denominators.

Ms. Hutchins: I also noticed the standard named creating common numerators and I wasn't sure what that meant. The way I learned to compare fractions was to find the least common multiple of the denominators to get common denominators. Like 12 is the least common multiple of 4 and 3 so to compare $\frac{3}{4}$ and $\frac{2}{3}$ you just multiply $\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$ and $\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$.

Let's Chat

- What do you notice and wonder about Ms. Hutchins analysis?

- How did the team discussion of the standard deepen their understanding?

Resource Spotlight

Louisiana STUDENT STANDARDS MATHEMATICS

Grade 4

Louisiana Student Standards: Companion Document for Teachers 2.0

This document is designed to assist educators in interpreting and implementing Louisiana's new mathematics standards. It contains descriptions of each grade 4 math standard to answer questions about the standard's meaning and how it applies to student knowledge and performance. Version 2.0 has been updated to include information from LDOE's Grade 4 Remediation and Rigor documents. Some examples have been added, deleted or revised to better reflect the intent of the standard. Examples are samples only and should not be considered an exhaustive list.

This companion document is considered a "living" document as we believe that teachers and other educators will find ways to improve the document as they use it. Please send feedback to LouisianaStandards@la.gov so that we may use your input when updating this guide.

Additional information on the Louisiana Student Standards for Mathematics, including how to read the standards' codes, a listing of standards for each grade or course, and links to additional resources, is available at http://www.louisianabelieves.com/resources/library/k-12-math-year-long-planning.

Updated November 7, 2019

DEPARTMENT of EDUCATION
louisiana believes

The Power of Progressions

Fragmenting the Standards into individual standards, or individual bits of standards, erases all these relationships and produces a sum of parts that is decidedly less than the whole.

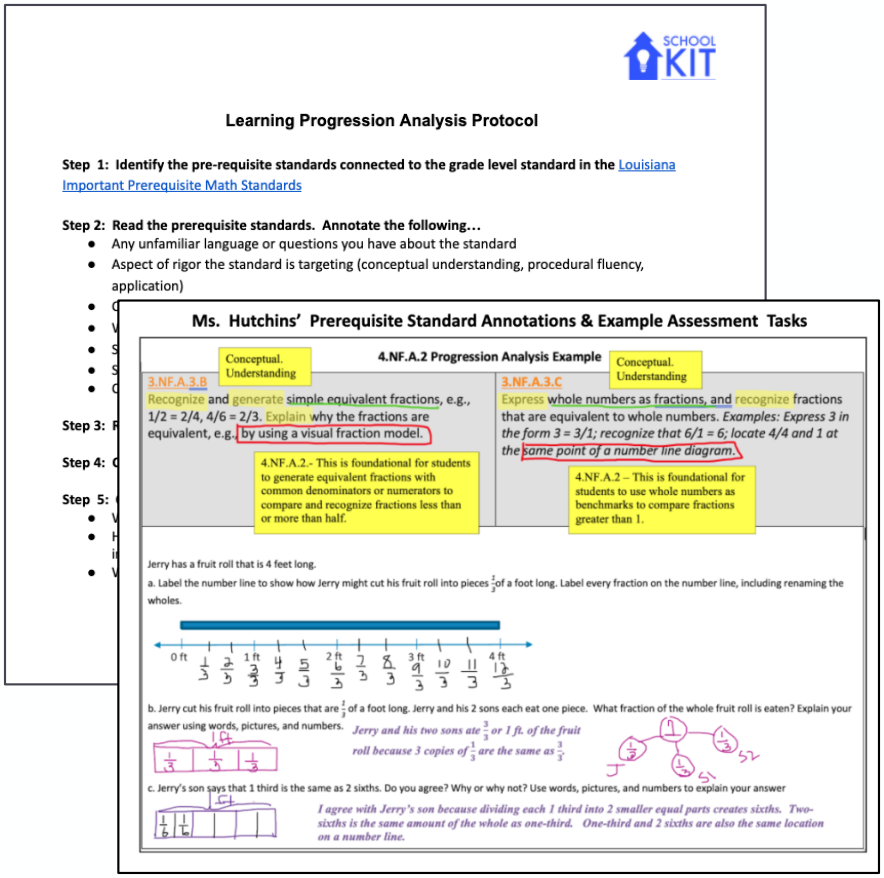
The standards were designed from **intentional progressions**. These progressions are important to help students learn higher mathematics.

Uncovering the Progressions

Review the Learning Progression Protocol and Ms. Hutchins' annotations of the pre-requisite standards.

Reflect & Discuss:

- How does this process for uncovering the progression of learning equip Ms.Hutchins to assess and diagnose unfinished learning?



SCHOOL KIT

Learning Progression Analysis Protocol

Step 1: Identify the pre-requisite standards connected to the grade level standard in the Louisiana Important Prerequisite Math Standards

Step 2: Read the prerequisite standards. Annotate the following…

- Any unfamiliar language or questions you have about the standard
- Aspect of rigor the standard is targeting (conceptual understanding, procedural fluency, application)

Step 3:

Step 4:

Step 5:

Ms. Hutchins' Prerequisite Standard Annotations & Example Assessment Tasks

4.NF.A.2 Progression Analysis Example

Conceptual. Understanding

3.NF.A.3.B

Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

4.NF.A.2.- This is foundational for students to generate equivalent fractions with common denominators or numerators to compare and recognize fractions less than or more than half.

Conceptual. Understanding

3.NF.A.3.C

Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*

4.NF.A.2 – This is foundational for students to use whole numbers as benchmarks to compare fractions greater than 1.

Jerry has a fruit roll that is 4 feet long.

a. Label the number line to show how Jerry might cut his fruit roll into pieces $\frac{1}{3}$ of a foot long. Label every fraction on the number line, including renaming the wholes.

0 ft, 1 ft, 2 ft, 3 ft, 4 ft

b. Jerry cut his fruit roll into pieces that are $\frac{1}{3}$ of a foot long. Jerry and his 2 sons each eat one piece. What fraction of the whole fruit roll is eaten? Explain your answer using words, pictures, and numbers. *Jerry and his two sons ate $\frac{3}{3}$ or 1 ft. of the fruit roll because 3 copies of $\frac{1}{3}$ are the same as $\frac{3}{3}$.*

c. Jerry's son says that 1 third is the same as 2 sixths. Do you agree? Why or why not? Use words, pictures, and numbers to explain your answer

I agree with Jerry's son because dividing each 1 third into 2 smaller equal parts creates sixths. Two-sixths is the same amount of the whole as one-third. One-third and 2 sixths are also the same location on a number line.

Resource Spotlight

NEBRASKA
DEPARTMENT OF EDUCATION

Essential Instructional Content for 2020-2021

Mathematics

This document has been adapted for use by the Nebraska Department of Education for Nebraska educators. The following guidance contains information about essential Mathematics content for the 2020-2021 school year.

Resource Spotlight

Considerations for Addressing ESSENTIAL Grade-Level Content	
The clusters and standards listed in this table name the essential instructional content for grade 4. The right-hand column contains approaches to shifting how time is dedicated to the clusters and standards in the left-hand column.	
Clusters/Standards	**Considerations**
See 2015 Nebraska College- and Career-Ready Mathematics Standards 4.1.2.h, 4.2.2.a, and 4.2.3.a. CCSSM; 4.OA.A	No special considerations for curricula well aligned to analyzing and solving multi-step word problems with the four operations (4.OA.3), and extending multiplicative thinking beyond grade 3 to solve problems involving comparison and the idea of times-as-many/times-as-much (4.OA.2).
See 2015 Nebraska College- and Career-Ready Mathematics Standards 4.1.1.a, 4.1.1.b, 4.1.1.f, and 4.1.1.g. CCSSM: 4.NBT.A	No special considerations for curricula well aligned to generalizing place value understanding, as detailed in this cluster. Time spent on instruction and practice should NOT be reduced.
See 2015 Nebraska College- and Career-Ready Mathematics Standards 4.1.1.i and 4.1.1.k. CCSSM: 4.NF.A	No special considerations for curricula well aligned to fraction equivalence and ordering, as detailed in this cluster. *Incorporate* some foundational work on simple equivalent fractions (3.NF.A.3). Time spent on instruction and practice should NOT be reduced.

Resource Spotlight

Considerations for Addressing ESSENTIAL Grade-Level Content	
The clusters and standards listed in this table name the essential instructional content for grade 4. The right-hand column contains approaches to shifting how time is dedicated to the clusters and standards in the left-hand column.	
Clusters/Standards	**Considerations**
See 2015 Nebraska College- and Career-Ready Mathematics Standards 4.1.2.h, 4.2.2.a, and 4.2.3.a. CCSSM; 4.OA.A	No special considerations for curricula well aligned to analyzing and solving multi-step word problems with the four operations (4.OA.3), and extending multiplicative thinking beyond grade 3 to solve problems involving comparison and the idea of times-as-many/times-as-much (4.OA.2).
See 2015 Nebraska College- and Career-Ready Mathematics Standards 4.1.1.a, 4.1.1.b, 4.1.1.f, and 4.1.1.g. CCSSM: 4.NBT.A	No special considerations for curricula well aligned to generalizing place value understanding, as detailed in this cluster. Time spent on instruction and practice should NOT be reduced.
See 2015 Nebraska College- and Career-Ready Mathematics Standards 4.1.1.i and 4.1.1.k. CCSSM: 4.NF.A	No special considerations for curricula well aligned to fraction equivalence and ordering, as detailed in this cluster. *Incorporate* some foundational work on simple equivalent fractions (3.NF.A.3). Time spent on instruction and practice should NOT be reduced.

Key Point

Uncovering the math and progressions in the standards, equips us to assess and accurately diagnose unfinished learning.

Our Agenda

1. Getting Started
2. Deepening Understanding of the Math We Teach
3. Assessing and Diagnosing Unfinished Learning
4. Reflection & Wrapping Up

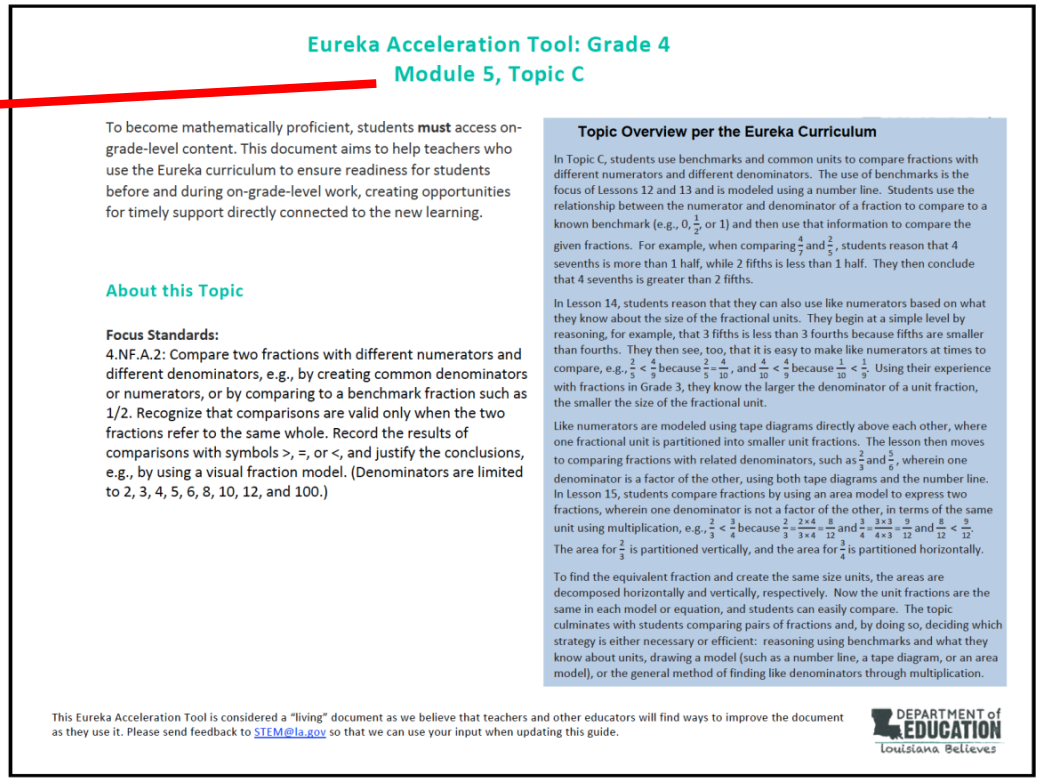
Key Actions to Diagnose

Identify	Identify the right diagnostics.
Consider	Consider what constitutes evidence.
Interpret	Interpret the evidence.

Resource Spotlight

Eureka Acceleration Tool

- Organized by Module (a "Module" is essentially a "unit" in the Eureka curriculum and Topic (a set of related lessons in the module)
- Includes a diagnostic assessment of prerequisite standards for the Topic (in this case, Comparing Fractions)



Eureka Acceleration Tool: Grade 4
Module 5, Topic C

To become mathematically proficient, students **must** access on-grade-level content. This document aims to help teachers who use the Eureka curriculum to ensure readiness for students before and during on-grade-level work, creating opportunities for timely support directly connected to the new learning.

About this Topic

Focus Standards:
4.NF.A.2: Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. (Denominators are limited to 2, 3, 4, 5, 6, 8, 10, 12, and 100.)

Topic Overview per the Eureka Curriculum

In Topic C, students use benchmarks and common units to compare fractions with different numerators and different denominators. The use of benchmarks is the focus of Lessons 12 and 13 and is modeled using a number line. Students use the relationship between the numerator and denominator of a fraction to compare to a known benchmark (e.g., 0, $\frac{1}{2}$, or 1) and then use that information to compare the given fractions. For example, when comparing $\frac{4}{7}$ and $\frac{2}{5}$, students reason that 4 sevenths is more than 1 half, while 2 fifths is less than 1 half. They then conclude that 4 sevenths is greater than 2 fifths.

In Lesson 14, students reason that they can also use like numerators based on what they know about the size of the fractional units. They begin at a simple level by reasoning, for example, that 3 fifths is less than 3 fourths because fifths are smaller than fourths. They then see, too, that it is easy to make like numerators at times to compare, e.g., $\frac{2}{5} < \frac{4}{9}$ because $\frac{2}{5} = \frac{4}{10}$, and $\frac{4}{10} < \frac{4}{9}$ because $\frac{1}{10} < \frac{1}{9}$. Using their experience with fractions in Grade 3, they know the larger the denominator of a unit fraction, the smaller the size of the fractional unit.

Like numerators are modeled using tape diagrams directly above each other, where one fractional unit is partitioned into smaller unit fractions. The lesson then moves to comparing fractions with related denominators, such as $\frac{2}{3}$ and $\frac{5}{6}$, wherein one denominator is a factor of the other, using both tape diagrams and the number line. In Lesson 15, students compare fractions by using an area model to express two fractions, wherein one denominator is not a factor of the other, in terms of the same unit using multiplication, e.g., $\frac{2}{3} < \frac{3}{4}$ because $\frac{2}{3} = \frac{2\times4}{3\times4} = \frac{8}{12}$ and $\frac{3}{4} = \frac{3\times3}{4\times3} = \frac{9}{12}$ and $\frac{8}{12} < \frac{9}{12}$. The area for $\frac{2}{3}$ is partitioned vertically, and the area for $\frac{3}{4}$ is partitioned horizontally.

To find the equivalent fraction and create the same size units, the areas are decomposed horizontally and vertically, respectively. Now the unit fractions are the same in each model or equation, and students can easily compare. The topic culminates with students comparing pairs of fractions and, by doing so, deciding which strategy is either necessary or efficient: reasoning using benchmarks and what they know about units, drawing a model (such as a number line, a tape diagram, or an area model), or the general method of finding like denominators through multiplication.

This Eureka Acceleration Tool is considered a "living" document as we believe that teachers and other educators will find ways to improve the document as they use it. Please send feedback to STEM@la.gov so that we can use your input when updating this guide.

DEPARTMENT of EDUCATION
Louisiana Believes

Eureka Acceleration Tools

- Currently available for **grades 4-8** (most major work topics)

What if we don't use Eureka or I teach a grade with no available Eureka Acceleration Tools?

- The process we are learning is **transferable** to any curriculum ⟶ focus on the key takeaways from the process!

Other Sources for Diagnostics

High Quality Curriculum

- Eureka Math Equip
- Previous Grade Level Tasks from Curriculum
- IM *Check Your Readiness*

Achieve the Core

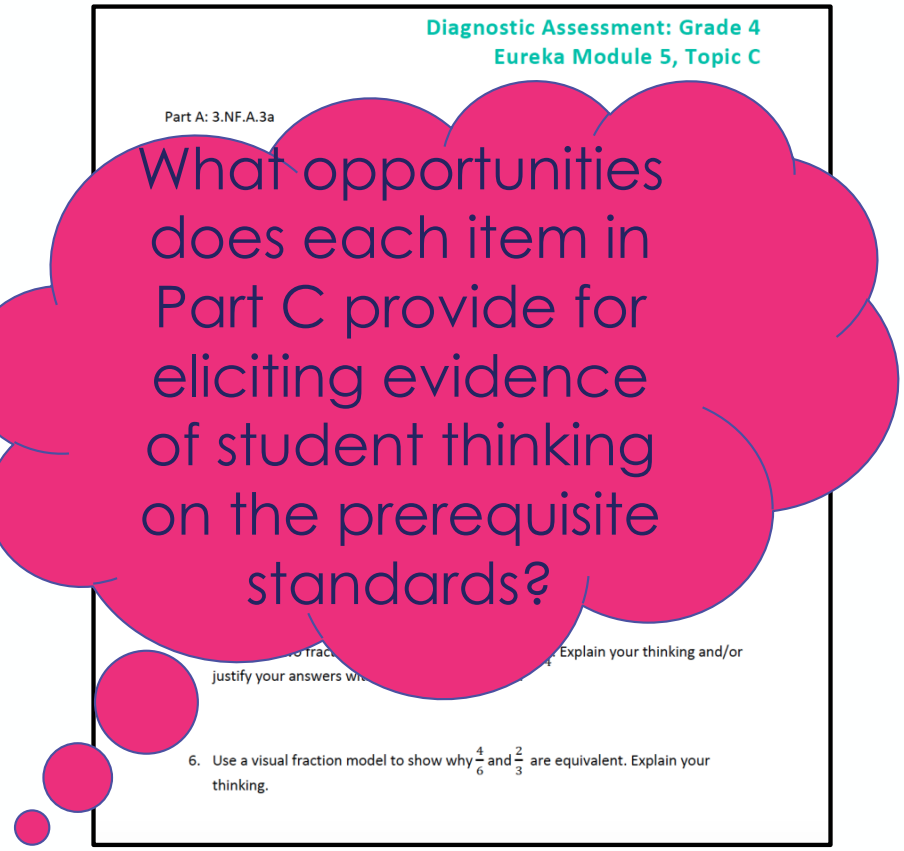
- Mini-assessments
- Do Nows for Unfinished Learning

Diagnostics are...

1. **TIMELY:** Happen at the unit/topic level or lesson level
2. **TARGETED:** Target the key prerequisite concepts/skills
3. **MANAGEABLE:** Can be administered without taking away from instructional time and provide a manageable amount of just in time data
4. **FORMATIVE:** Assessment FOR learning, not Assessment OF learning; Used to adjust instruction, not sort students based on perceived readiness

Zoom In: Diagnostic Assessment

- Organized into 2-3 parts (A, B, C)
- Each part is aligned to a foundational standard from previous grade
- Each part has three items



Key Actions to Diagnose

Identify	Identify the right diagnostics.
Consider	Consider what constitutes evidence.
Interpret	Interpret the evidence.

Define what Constitutes Evidence

Part C Focus: 3.NF.A.3d. Explain equivalence of fractions with denominators 2, 3, 4, 6, and 8 in special cases, and compare fractions by reasoning about their size.

d) Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Why this is important for current grade level work:
Comparing fractions with the same numerator or the same denominator sets the foundation for comparing fractions with different numerators and different denominators, the sole focus of the target topic. Visual models also set the foundation for comparing fractions with the general method of finding equivalent fractions later.

Using the Diagnostic Assessment to identify gaps:

Problem 7:
Students must understand that for a comparison to be valid each fraction must refer to the same whole. Furthermore, it is important that students understand that the denominator does not dictate the whole, only the number of parts that comprise a whole.

Problems 8-9:
Look for students who only compare the numbers that differ and treat them as whole numbers. This will be more evident in Problem 9 If a student thinks 2/6 is less than 2/8 because 6 is less than 8. Encourage the use of visual fraction models and look for models that show equal wholes.

Remediation Resources for Targeted Instruction:

3rd Grade, Module 5, Topic F, Lesson(s) 28 - 29

Use the Concept Development portion of each Lesson and a sampling of problems from the Problem Set focused on conceptual understanding.

Review the look fors for Part C in the Topic C Diagnostic Assessment Tasks (pg. 7)

Define what Constitutes Evidence

- Create exemplar response for **ONE** of the items in Part C

- Based on Ms. Hutchins standard and progression analysis, what misconceptions and incomplete understandings might the item reveal?

Diagnostic Assessment: Grade 4
Eureka Module 5, Topic C

Part C: 3.NF.A.3d

7. For the inequality $\frac{1}{2} > \frac{1}{4}$ to be valid, what must be true?

8. Complete the sentence with >, =, or <. Explain your thinking and/or justify your choice with a visual fraction model.

   $\frac{2}{6}$ _____ $\frac{5}{6}$

9. Complete the sentence with >, =, or <. Explain your thinking and/or justify your choice with a visual fraction model.

   $\frac{2}{6}$ _____ $\frac{2}{8}$

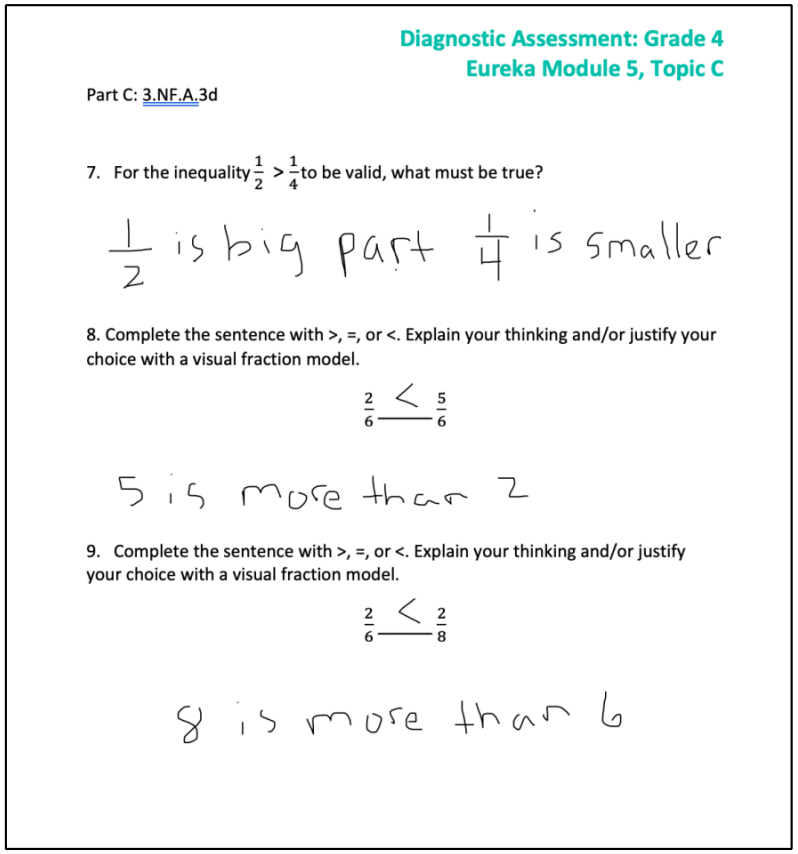
Key Actions to Diagnose

Identify	Identify the right diagnostics.
Consider	Consider what constitutes evidence.
Interpret	Interpret the evidence.

Interpret the Evidence

Examine the student work samples.

- What stands out to you about the teacher analysis?
- How does the previous work uncovering the standards, and learning progression inform the interpretation of student work?



Diagnostic Assessment: Grade 4
Eureka Module 5, Topic C

Part C: 3.NF.A.3d

7. For the inequality $\frac{1}{2} > \frac{1}{4}$ to be valid, what must be true?

$\frac{1}{2}$ is big part $\frac{1}{4}$ is smaller

8. Complete the sentence with >, =, or <. Explain your thinking and/or justify your choice with a visual fraction model.

$\frac{2}{6} \_\_ \frac{5}{6}$

5 is more than 2

9. Complete the sentence with >, =, or <. Explain your thinking and/or justify your choice with a visual fraction model.

$\frac{2}{6} \_\_ \frac{2}{8}$

8 is more than 6

When we look at student work...

Evidence vs. Inference

EVIDENCE looks like…

- Pictures and diagrams
- Calculations
- Justification

EVIDENCE of student understanding creates…

- Conclusions with confidence
- Focused, intentional next steps

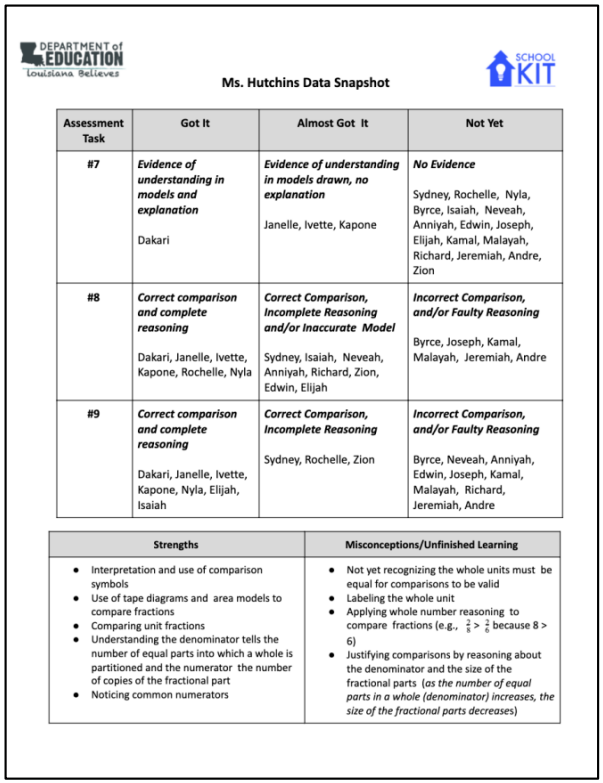
INFERENCE sounds like…

- "He does it in class."
- "It was a careless mistake."
- "He was having a bad day."

INFERENCE of student understanding creates…

- Misdiagnosis or Premature advance
- Apparent difficulty with retention of topics

Interpret the Evidence



DEPARTMENT of EDUCATION Louisiana Believes

SCHOOL KIT

Ms. Hutchins Data Snapshot

Assessment Task	Got It	Almost Got It	Not Yet
#7	***Evidence of understanding in models and explanation*** Dakari	***Evidence of understanding in models drawn, no explanation*** Janelle, Ivette, Kapone	***No Evidence*** Sydney, Rochelle, Nyla, Byrce, Isaiah, Neveah, Anniyah, Edwin, Joseph, Elijah, Kamal, Malayah, Richard, Jeremiah, Andre, Zion
#8	***Correct comparison and complete reasoning*** Dakari, Janelle, Ivette, Kapone, Rochelle, Nyla	***Correct Comparison, Incomplete Reasoning and/or Inaccurate Model*** Sydney, Isaiah, Neveah, Anniyah, Richard, Zion, Edwin, Elijah	***Incorrect Comparison, and/or Faulty Reasoning*** Byrce, Joseph, Kamal, Malayah, Jeremiah, Andre
#9	***Correct comparison and complete reasoning*** Dakari, Janelle, Ivette, Kapone, Nyla, Elijah, Isaiah	***Correct Comparison, Incomplete Reasoning*** Sydney, Rochelle, Zion	***Incorrect Comparison, and/or Faulty Reasoning*** Byrce, Neveah, Anniyah, Edwin, Joseph, Kamal, Malayah, Richard, Jeremiah, Andre

Strengths	Misconceptions/Unfinished Learning
• Interpretation and use of comparison symbols • Use of tape diagrams and area models to compare fractions • Comparing unit fractions • Understanding the denominator tells the number of equal parts into which a whole is partitioned and the numerator the number of copies of the fractional part • Noticing common numerators	• Not yet recognizing the whole units must be equal for comparisons to be valid • Labeling the whole unit • Applying whole number reasoning to compare fractions (e.g., $\frac{1}{8} > \frac{1}{6}$ because 8 > 6) • Justifying comparisons by reasoning about the denominator and the size of the fractional parts (*as the number of equal parts in a whole (denominator) increases, the size of the fractional parts decreases*)

Equips her to identify…

- student strengths to connect and build on
- specific models students are currently using and their level of precision with those models
- specific concepts students have unfinished learning with (e.g., the size of the whole units must be the same when comparing fractions)
- specific misconceptions to address in instruction (applying whole number reasoning to compare fractions)
- concepts to target in whole group instruction and small group instruction
- students for targeted small group instruction and/or individual interventions

Key Points

To accurately diagnose unfinished learning, we must:

- look at **bite-sized amounts** of **"just in time"** data (formative data at the topic or even lesson level)
- **Interpret evidence** of student learning and identify specifically what students currently do understand/can do and what they don't yet understand/have the ability to do

Our Agenda

1. Getting Started
2. Deepening Understanding of the Math We Teach
3. Assessing and Diagnosing Unfinished Learning
4. Reflection & Wrapping Up

Let's Reflect

Independently Reflect:

- To what extent is this work currently happening at your school/in your classroom?

- What implications might this learning have on how you support schools or teachers with assessing and diagnosing unfinished learning in your role?

Looking Ahead

Session 1	Session 2	Session 3	Session 4
Defining our Approach to Addressing Unfinished Teaching and Learning in Math	Assessing and Diagnosing Unfinished Learning in Math	Plan and Take Action Part I: Planning Intentional Core Supports	Plan and Take Action Part II: Planning Intentional Small Group and Individual Supports